MHD Simulations of the Jet in the Crab Nebula

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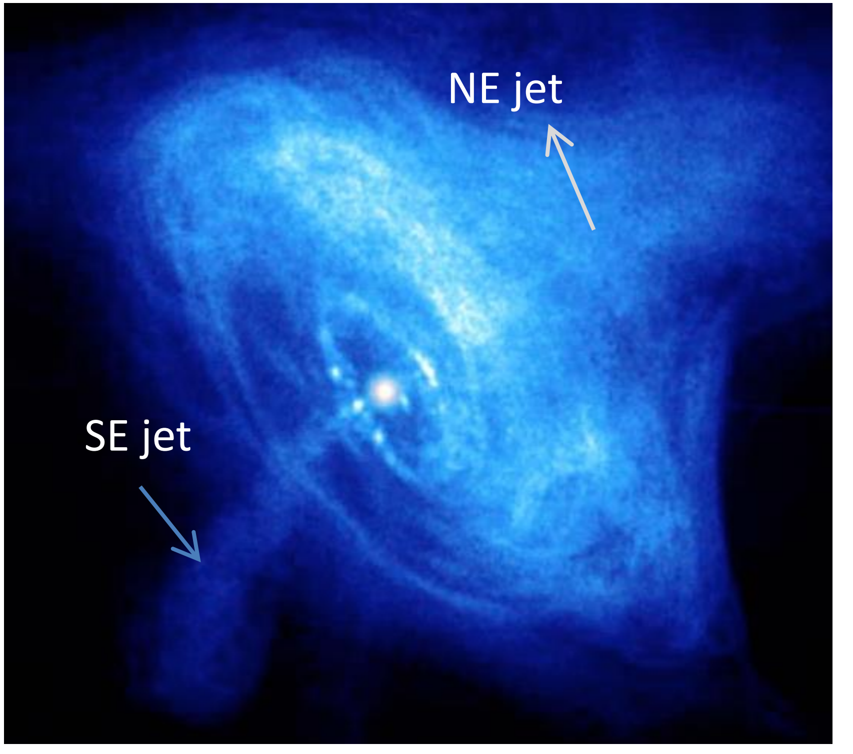
Outline

1. *Observational Evidence*
2. *Numerical Models of Relativistic MHD jets*
3. *Results*
4. *Summary*

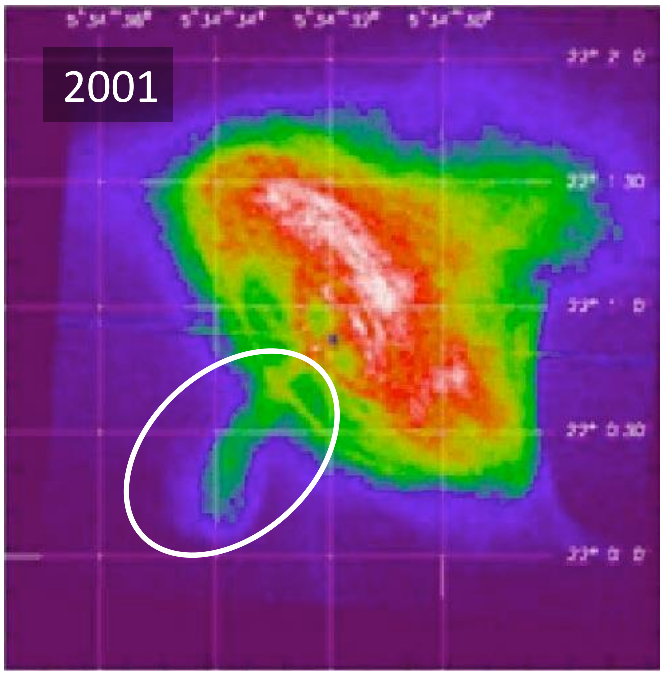
Observational Evidence

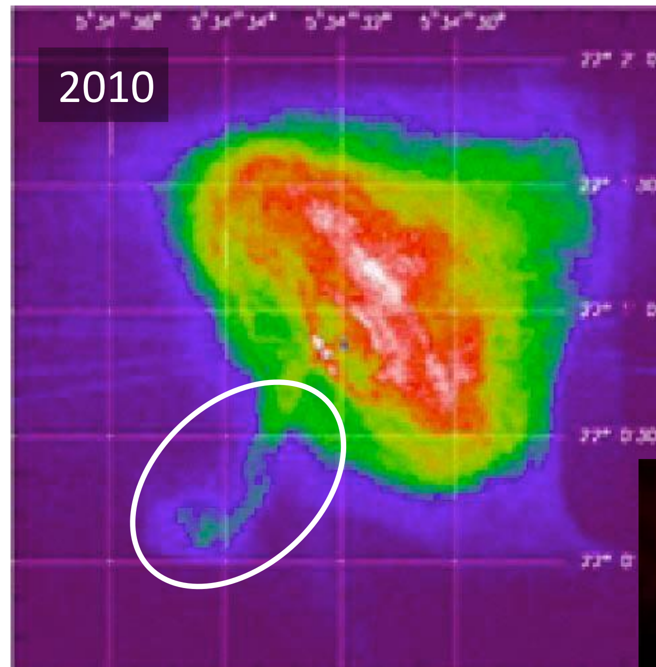
- X-ray observation (Chandra) show the emergence of a bipolar jets and extending to the SE and NEW of the pulsar
- A region of diffuse emission (anvil) may be associated with shocks and marks the base of the X-ray and optical jet
- Knots of emission are seen along the jets
- In the SE jet material flows with v/c~0.4 slowing down to ~0.02 into the nebula



Jet Wiggling

- SE jet morphology is "S" shaped and show remarkable time variability (Weisskopf)

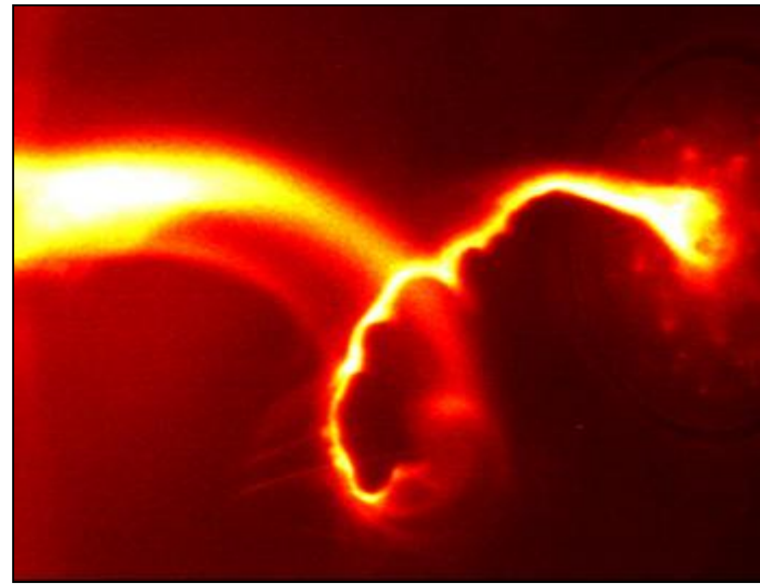




- Jet wiggling in other PWN, Vela (Durant 2013)
- → evidence for some kind of (intrinsic) flow instability

Jet instability in laboratory

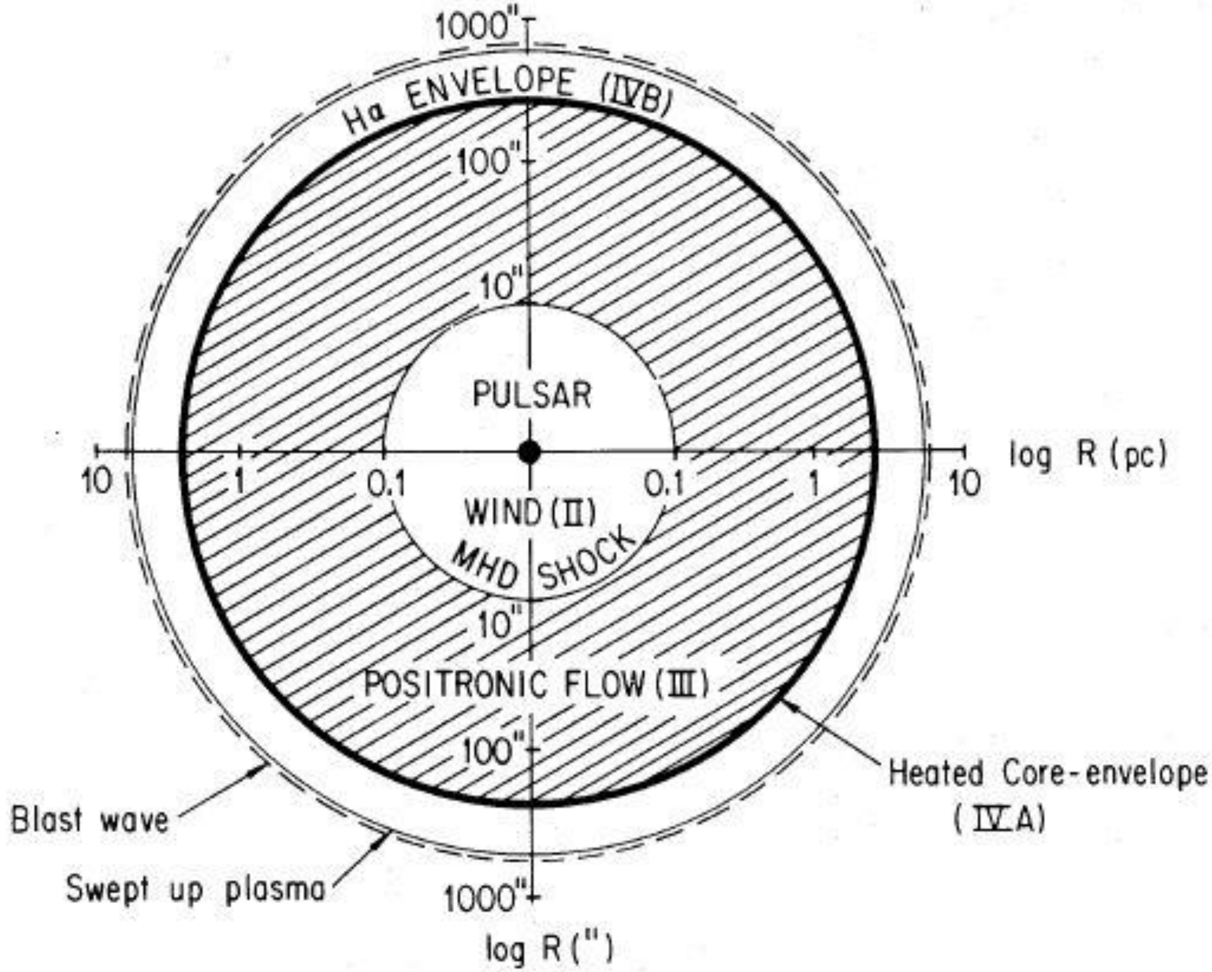
1. Magnetic field reconnection in "islands" related to kink instabilities
2. Reconnection detected in tomakaks as "sawthooth oscillations" and/or runaway acceleration
3. Particle acceleration in kink-driven reconnection events
4. A framework for the Crab gamma-ray flares originating in the "anvil" region.



A.L. Moser, P. Bellan, Nature , 482, 379 (2012)

Kennel–Coroniti picture of the Crab Nebula

1984



Pulsar Wind Model

MHD termination shock

PSR wind magnetization $$s = \frac{B^2}{4pnugmc^2}$$

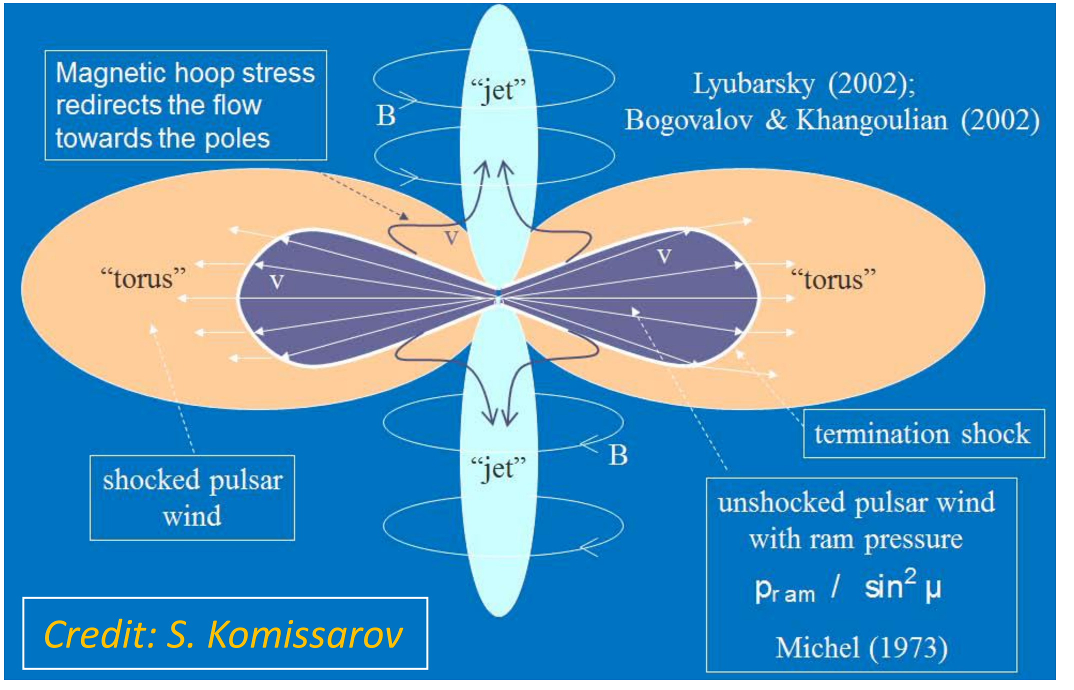
KC solution in the toroidal shock: $\sigma \leq 0.01$

The *sigma-problem*: large magnetic field required for the acceleration up to gamma-ray energies, low magnetic field in the shock region

Origin of the Jet

- Jet forms downstream of the wind termination shock

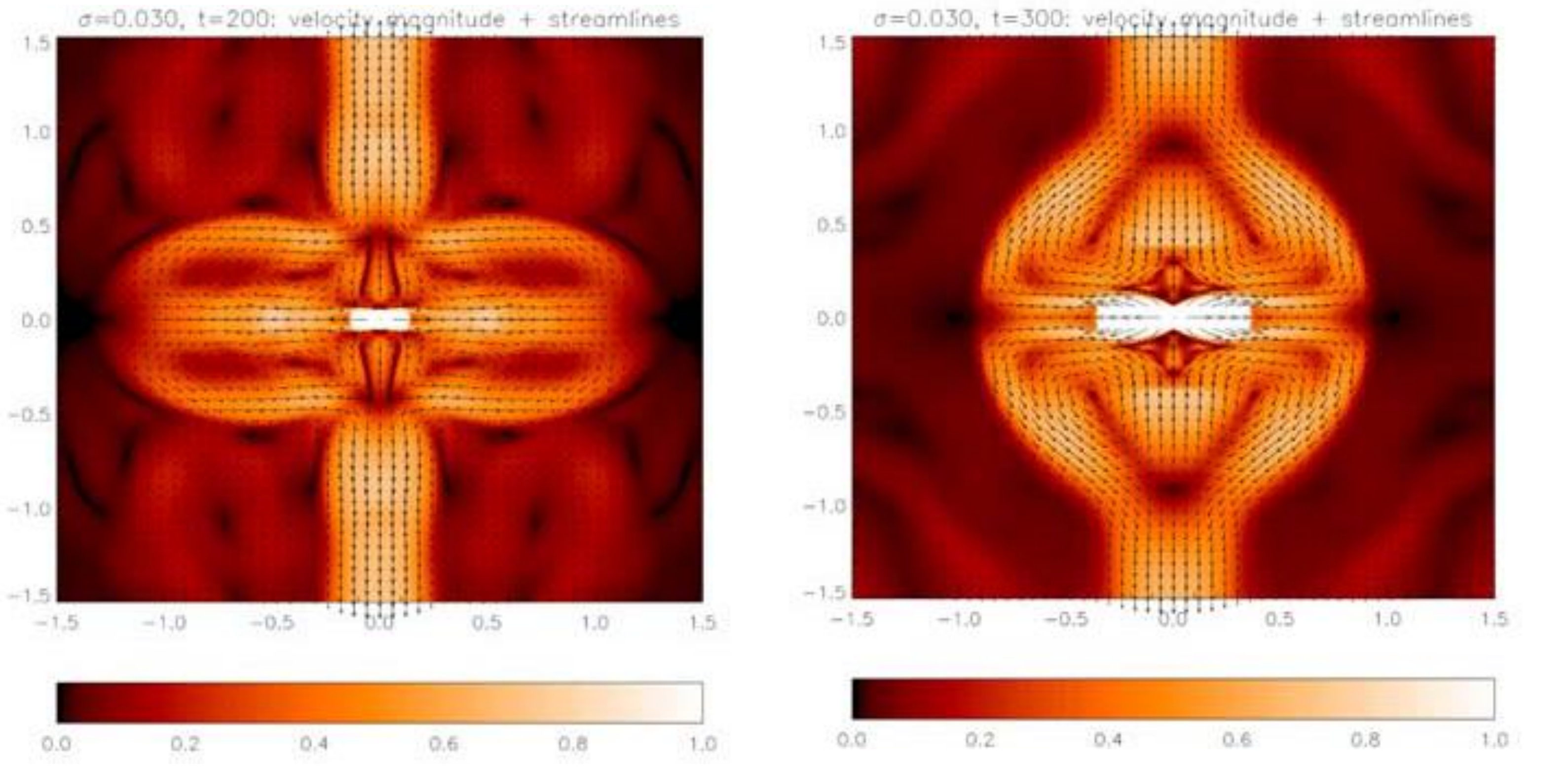
- Magnetic fields confine matter towards polar axis
  → "***tooth-paste***" effect:
  hoop stress of the azimuthal magnetic field carried by the wind (Lyubarsky 2002)



- Models confirmed by 2D axisymmetric numerical simulations (Komissarov & Lyubarski 2003,2004, Del Zanna et al. 2004, Bogovalov et al. 2005)

Jet Origin: previous results

- 2D MHD simulations (do not allow smal pitch azimuthal perturbations)
- For moderate/large $\sigma = B^2/(8\pi\rho c^2\gamma^2)$ magnetic hoop stress suppresses high velocity outflows in the equatorial plane and divert them towards the polar axis partially driving the super-fast jet[1]



[1]Del Zanna et al, A&A (2004) 421,1063

A 3D MHDR Jet Model

- We solve the equations for a relativistic perfectly conducting fluid describing energy/momentum and particle conservation (relativistic MHD equations)

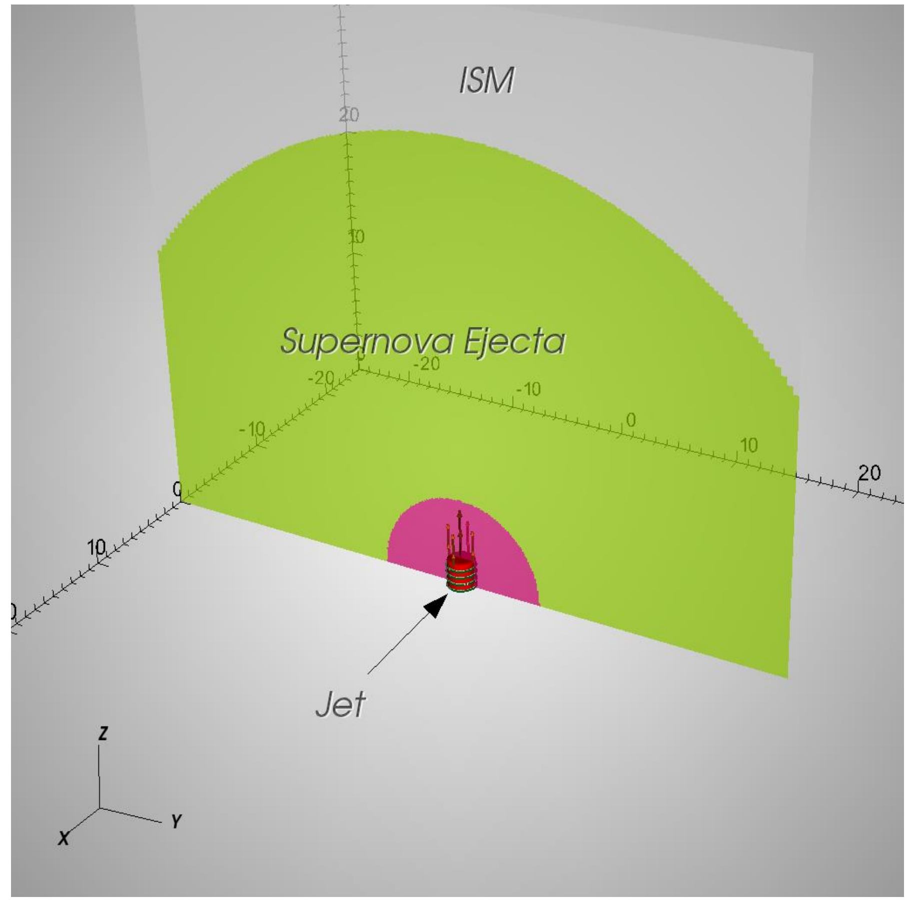
$$
\begin{aligned}
\frac{\partial}{\partial t}(\rho\gamma) + \nabla\cdot(\rho\gamma\mathbf{v}) &= 0 \\
\frac{\partial \mathbf{m}}{\partial t} + \nabla\cdot\left[w\gamma^2\mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} - \mathbf{E}\mathbf{E}\right] + \nabla p_t &= 0 \\
\frac{\partial \mathbf{B}}{\partial t} - \nabla\times(\mathbf{v}\times\mathbf{B}) &= 0 \\
\frac{\partial \mathcal{E}}{\partial t} + \nabla\cdot(\mathbf{m} - \rho\gamma\mathbf{v}) &= 0
\end{aligned}
$$

$$
\mathcal{E} = w\gamma^2 - p + \frac{\mathbf{B}^2 + \mathbf{E}^2}{2} - \rho\gamma
$$

- We use the PLUTO[1,2] code for astrophysical fluid dynamics
  (freely distributed *http://plutocode.ph.unito.it*)
- Numerical resolution 320 x 320 x 768 zones (≈ 20 point on the jet)

Numerical Setup

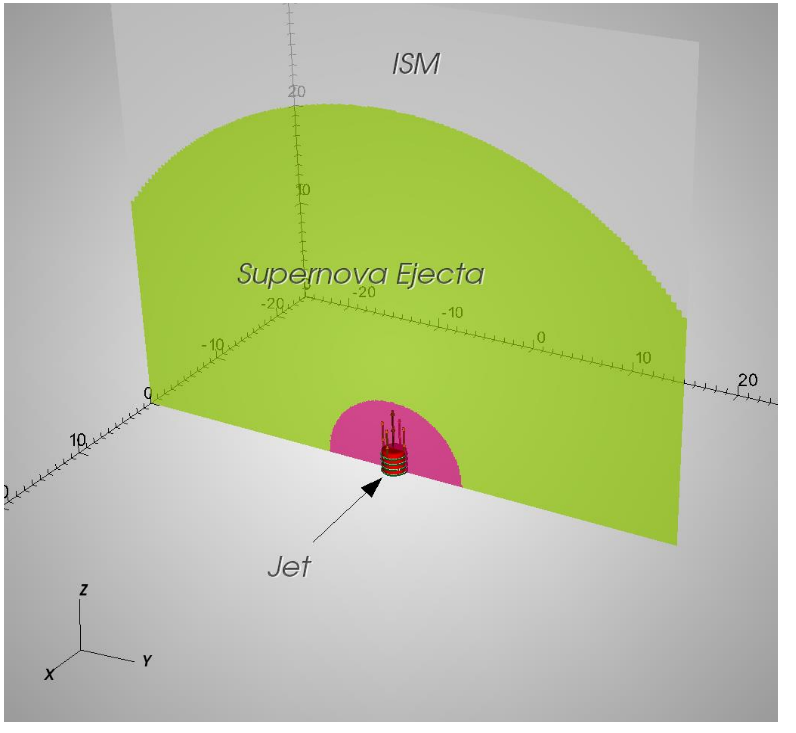
- Initial conditions from Del Zanna et al. (2004)
- Inside 0.2 < r < 1 (ly): freely expanding supernova ejecta (3 M_{sun}, E = 10^{51} erg) self-similar velocity increasing with *r*
- Jet enters at the lower *z* boundary
- Pulsar wind structure not modelled: assume jet already formed as the result of the collimation process
- Jet radius $R_j = 3 \times 10^{16}$ cm
- Computational domain:
  $x,y \in [-25,25]\ R_j/c$, $z \in [0, 80]\ R_j/c$;
  ($\approx 1.6 \div 2.5$ ly)



Model Parameters

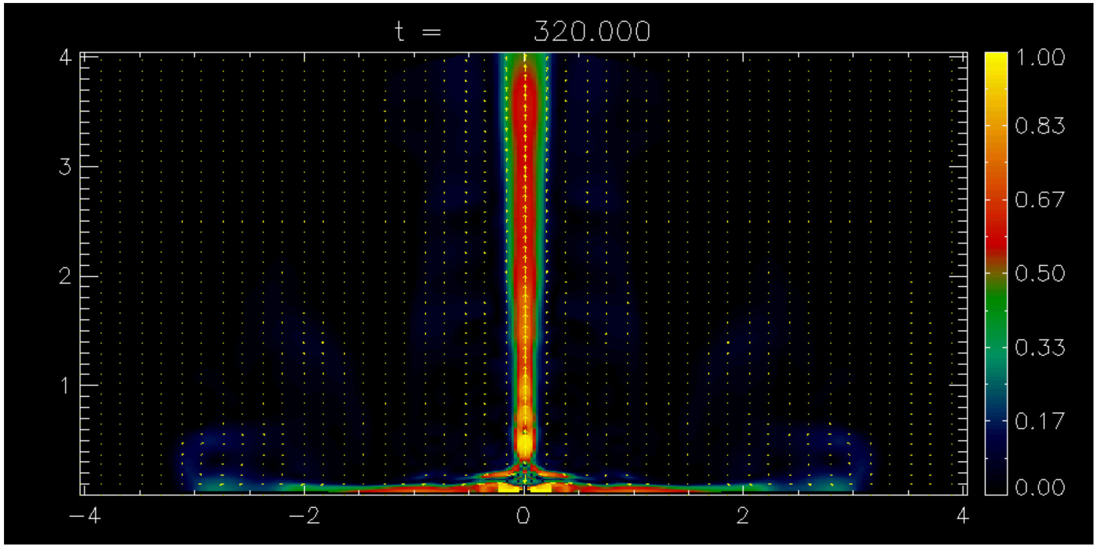
- Jet flow modeled by 5 parameters:

1. Sonic flow Mach number: $M_s = v_j / c_s$
2. Bulk Lorentz factor: $\gamma_j = (1 - v^2_j)^{-1/2}$
3. Jet/ambient dens. contrast: $\eta = \rho_j / \rho_e$
4. Magnetization: $\sigma = B^2 / (8\pi\rho\gamma^2)$;
5. Pitch angle: $P = RB_z / B_\phi$



Parameter Constraints

- Parameters are fixed through the 2D axisymmetric results
  - $1.3 \leq M_s \leq 2$ → hot jet
  - $2 \leq \gamma \leq 4$
  - σ = ?
  - Density contrast $\eta = 10^{-6}$
  - Azimuthal field implies Pitch $\rightarrow 0$ ($B_z = 0$)



- We consider hollow ($\eta = 10^{-6}$), hot ($M_s = 1.7$) jets initially carrying purely axial current ($B_\phi > 0$, $B_z = B_R = 0$)
- $p(R)$, $B_\phi(R)$ are set by radial momentum balance across the jet
- This leaves γ and σ as free parameters

Simulation Cases

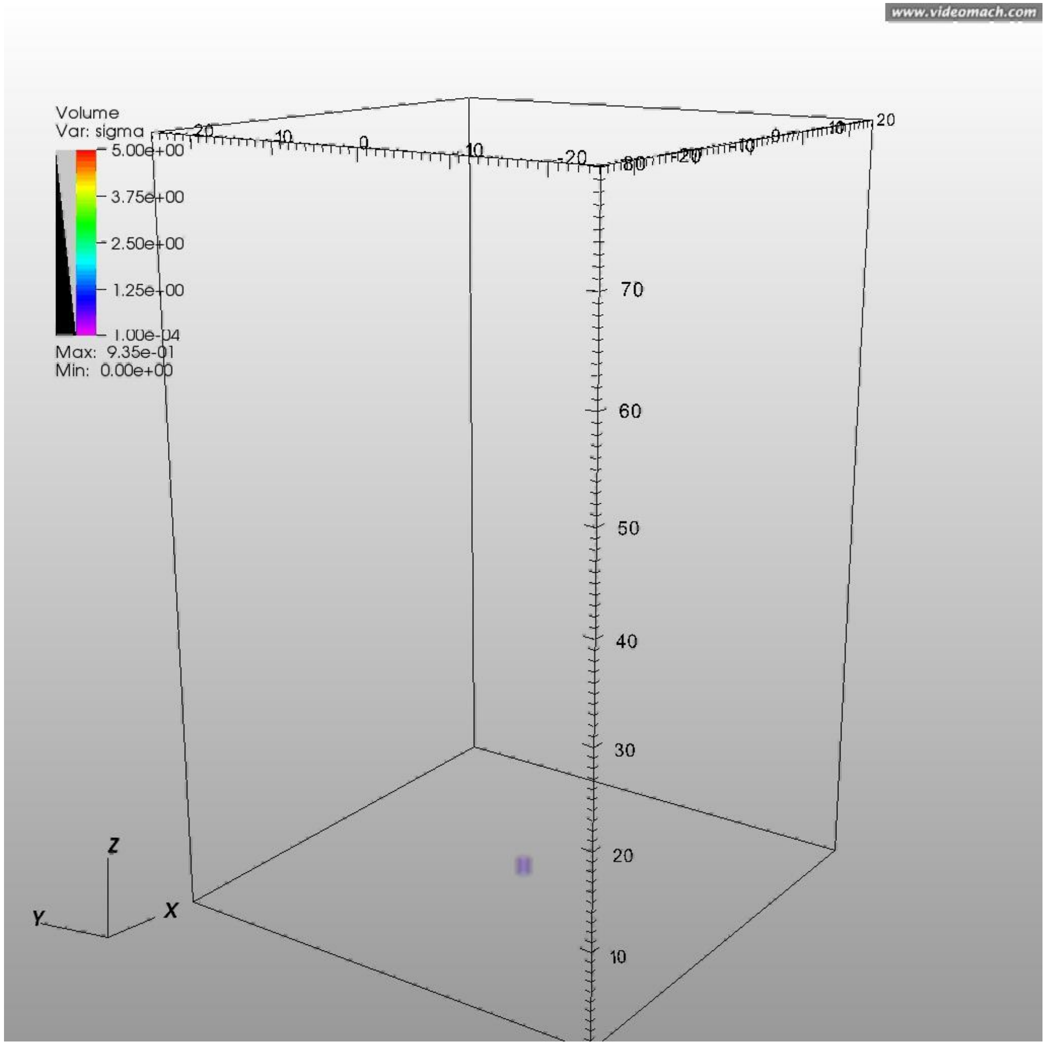
- We explore different values of Lorentz factor γ (= 2, 4) and magnetization σ (= 0.1, 1, 10) for a total of 6 different cases:

Case	γ	σ	*Plasma* β
A1	2	0.1	*4.5*
A2	2	1	*0.6*
A3	2	10	*0.2*
B1	4	0.1	*11.4*
B2	4	1	*1.2*
B3	4	10	*0.15*

- Random perturbations are applied of helical and fluting types at high and low frequencies

Results: Case A2

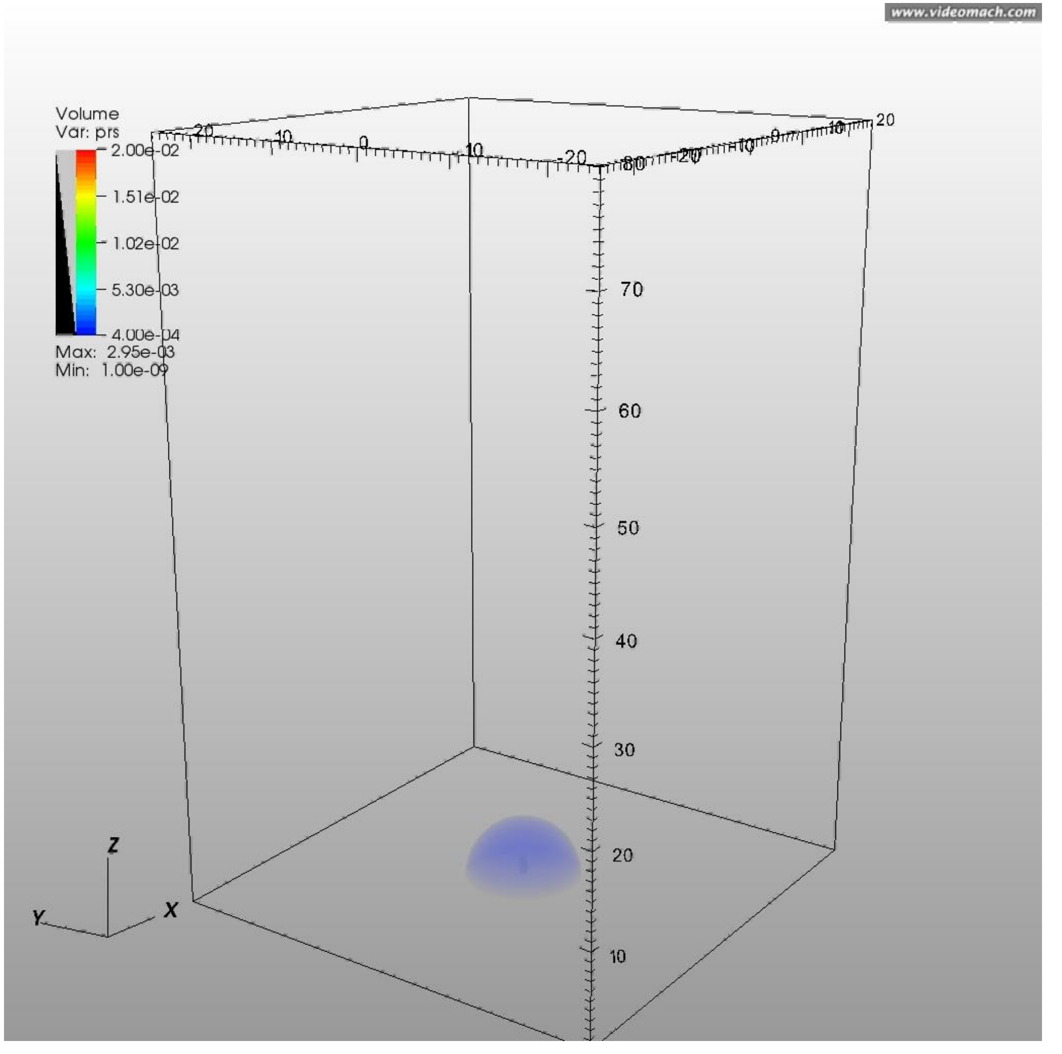
Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10



Sigma distribution

Results: Case A2

Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

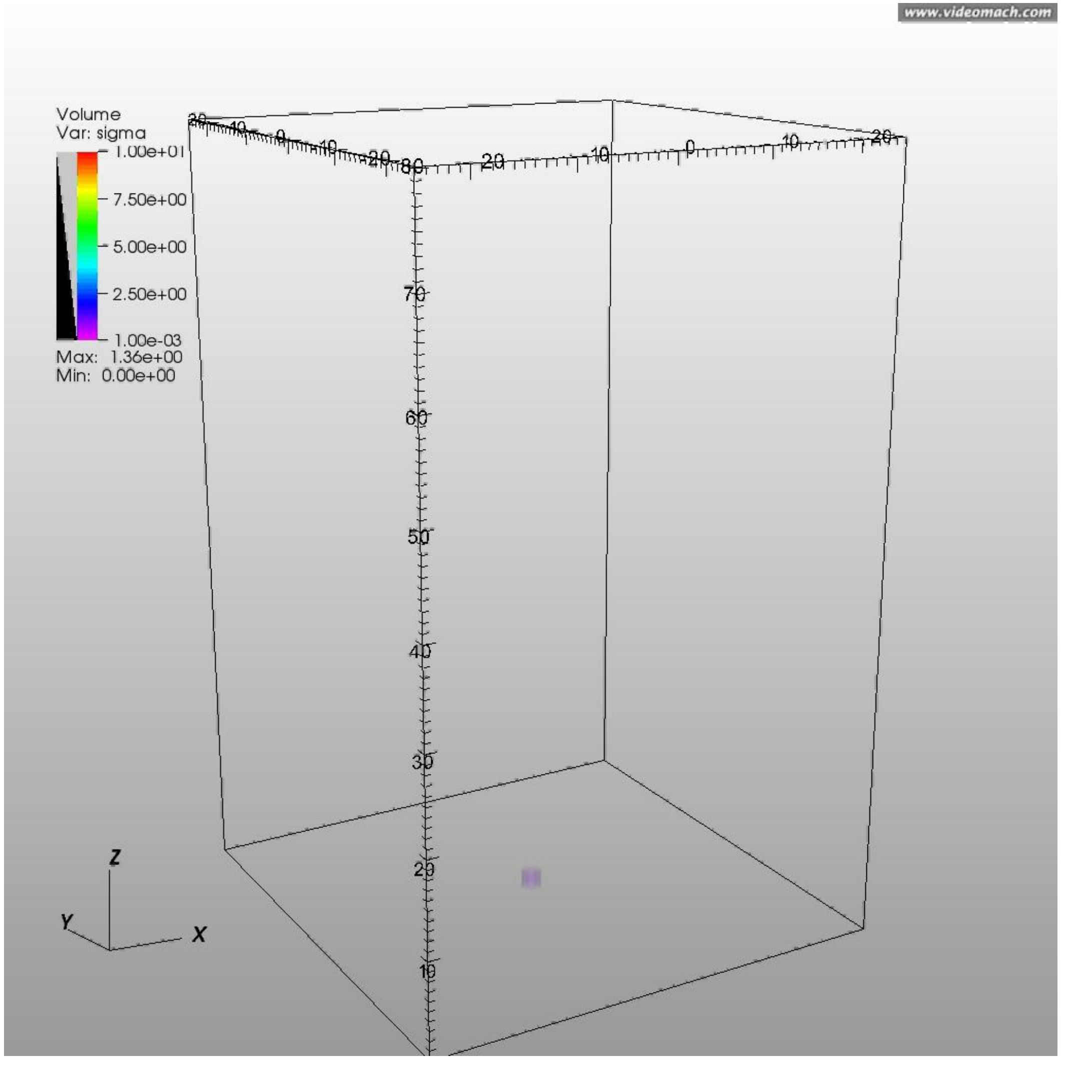


Pressure distribution

Results: Case B2

Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

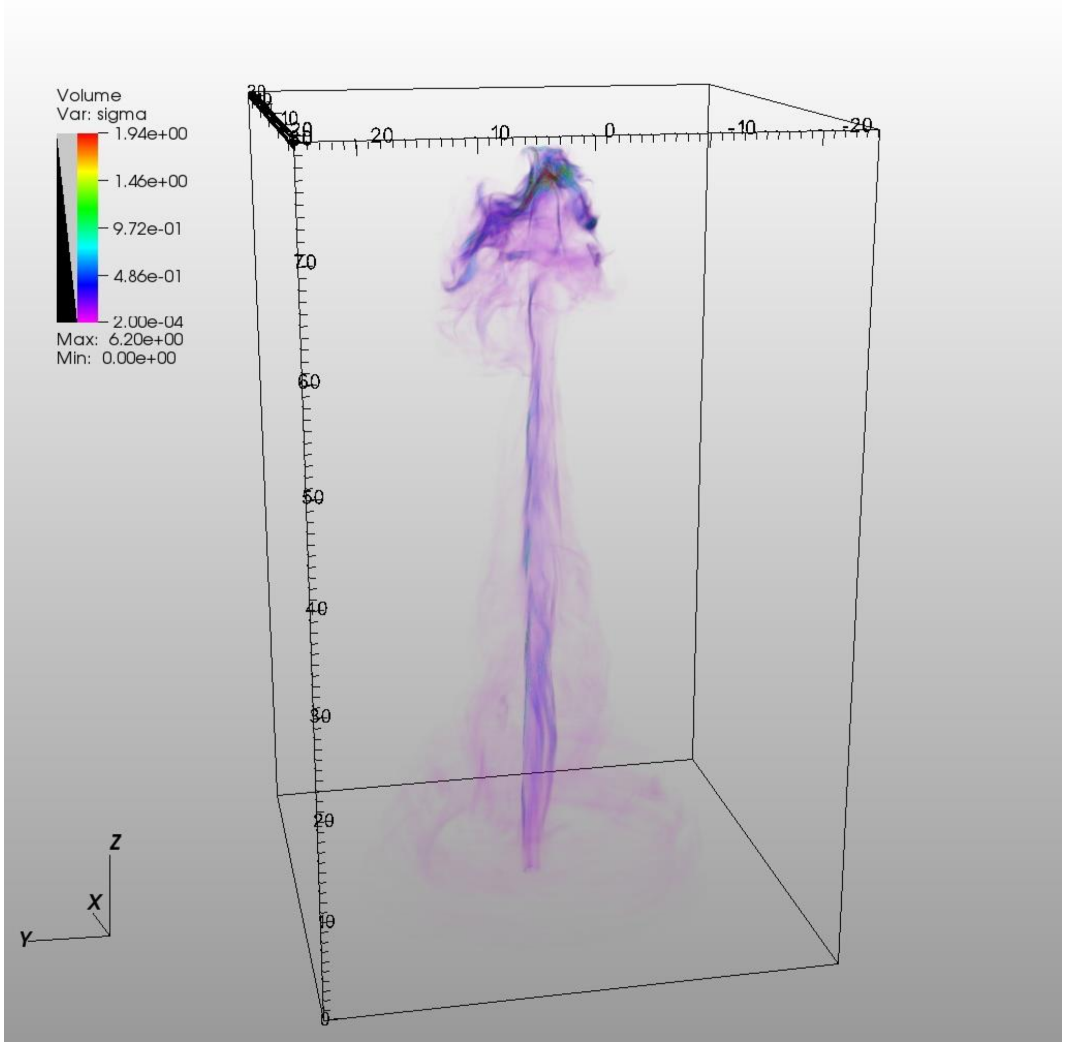
Sigma distribution



Results: Case B1

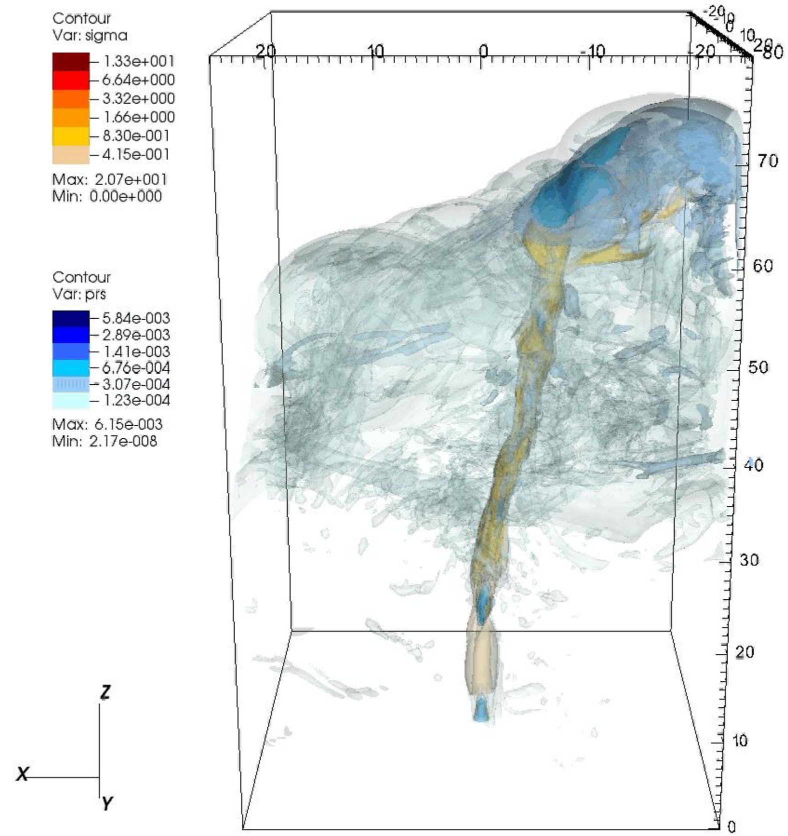
Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

<u>*Sigma distribution*</u>

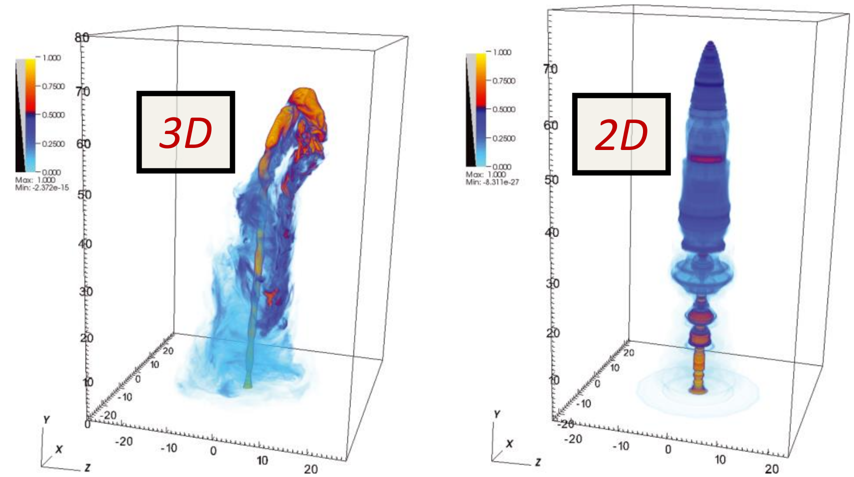


General Features

- Jets have small propagation speed (0.02c – 0.08c)
- Large over-pressurized turbulent cocoons
- Collimated central spines moving at mildly relativistic speeds
- Cocoon less magnetized than central spine
- Large-scale deflections may be present



General Features



- 3D models very different from 2D counterparts[1]:

- Strong toroidal configurations expected to become unstable to current driven modes. Most unstable mode m=1 (kink)

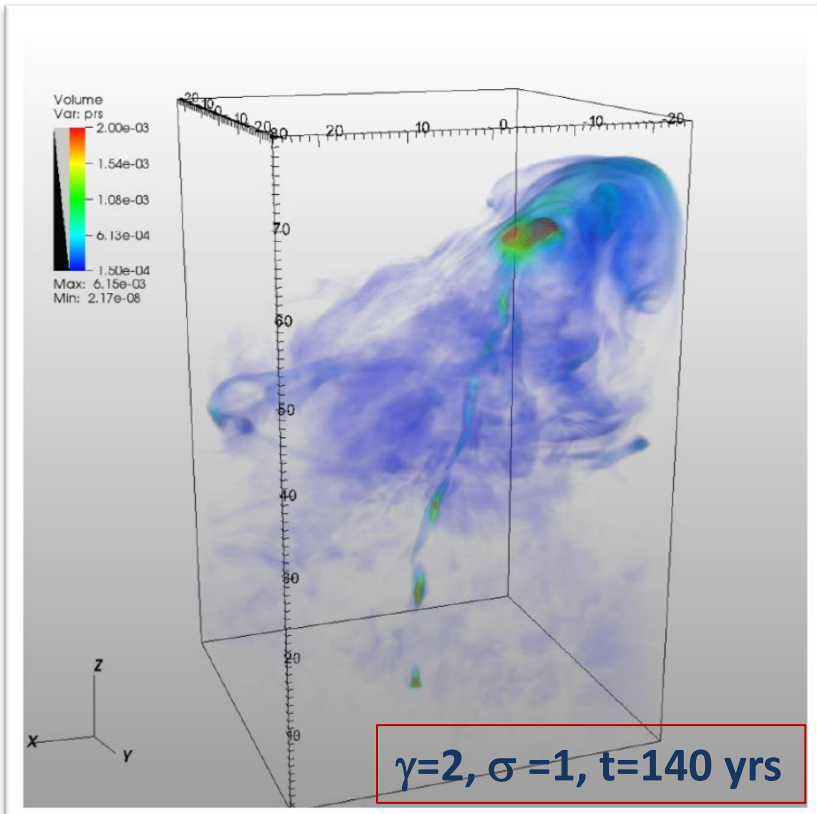
- Jet develops non-axisymmetric structures with large time-dependent deflections off the longitudinal axis

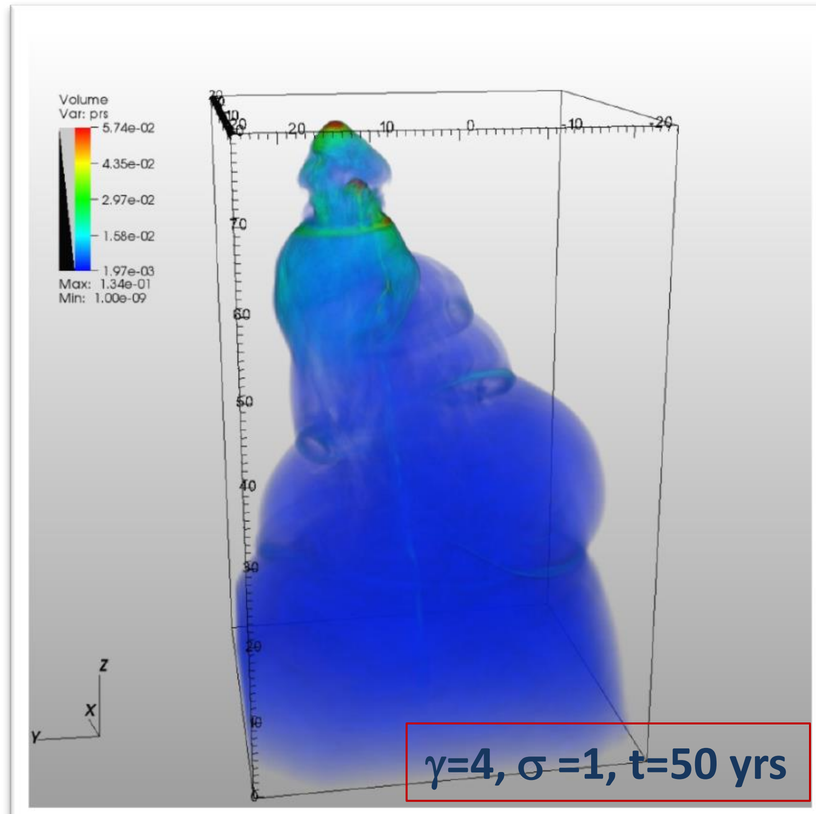
- Deflection time-scale of the order of a few years

[1]Mignone et al. MNRAS (2010), 402, 7

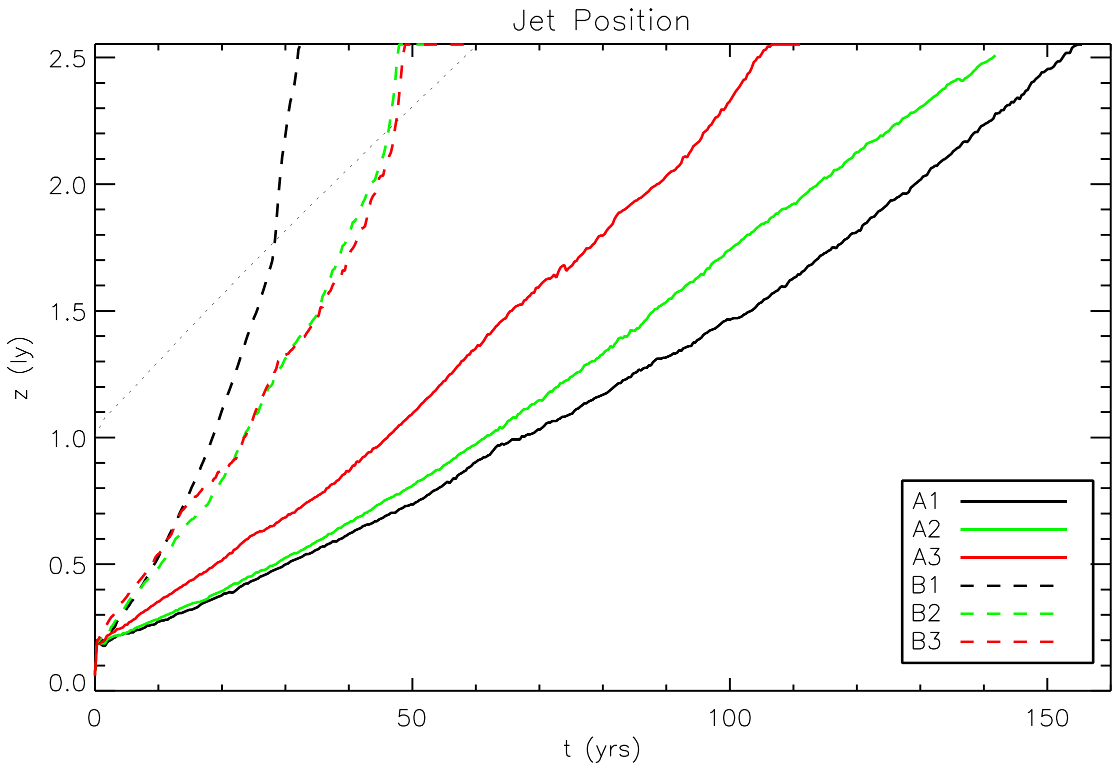
General Features

- Wiggling and deflection more pronounced at the terminal bow shock where magnetic field is amplified:





Jet Head Position



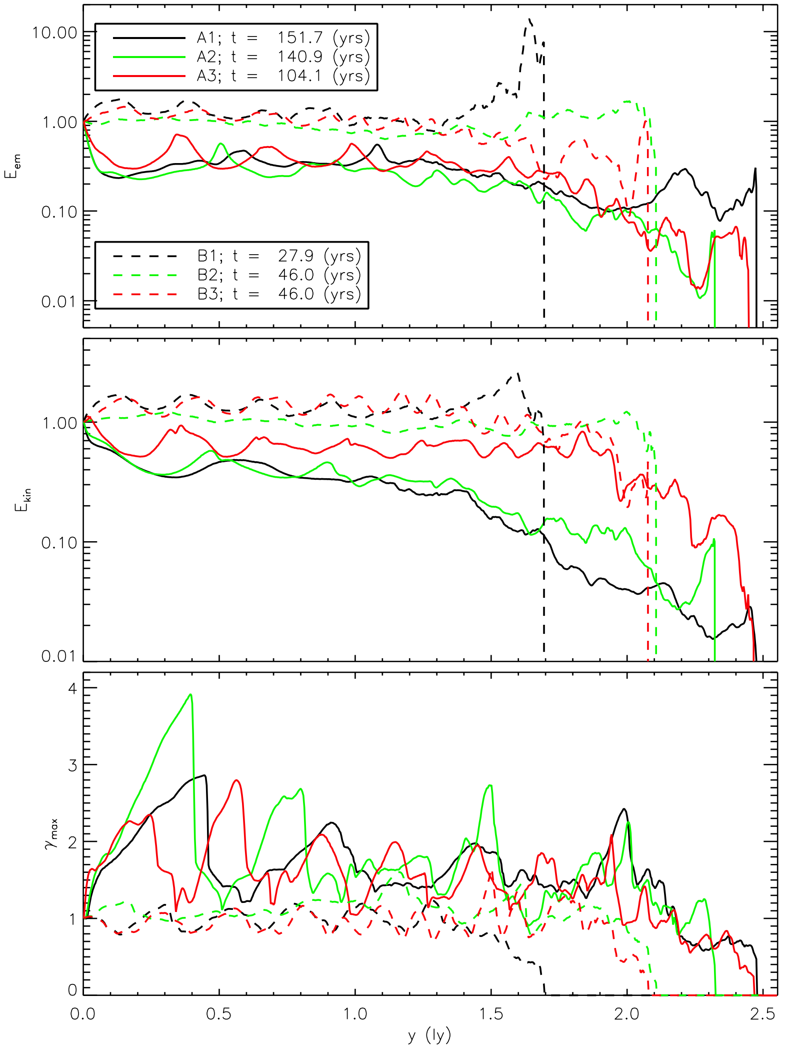
- Jets are slow because of large density contrast ($\rho_j / \rho_e < 10^{-6}$)
- Faster jets reach the outer edge of the expanding nebula

Magnetic energy *vs* kinetic energy

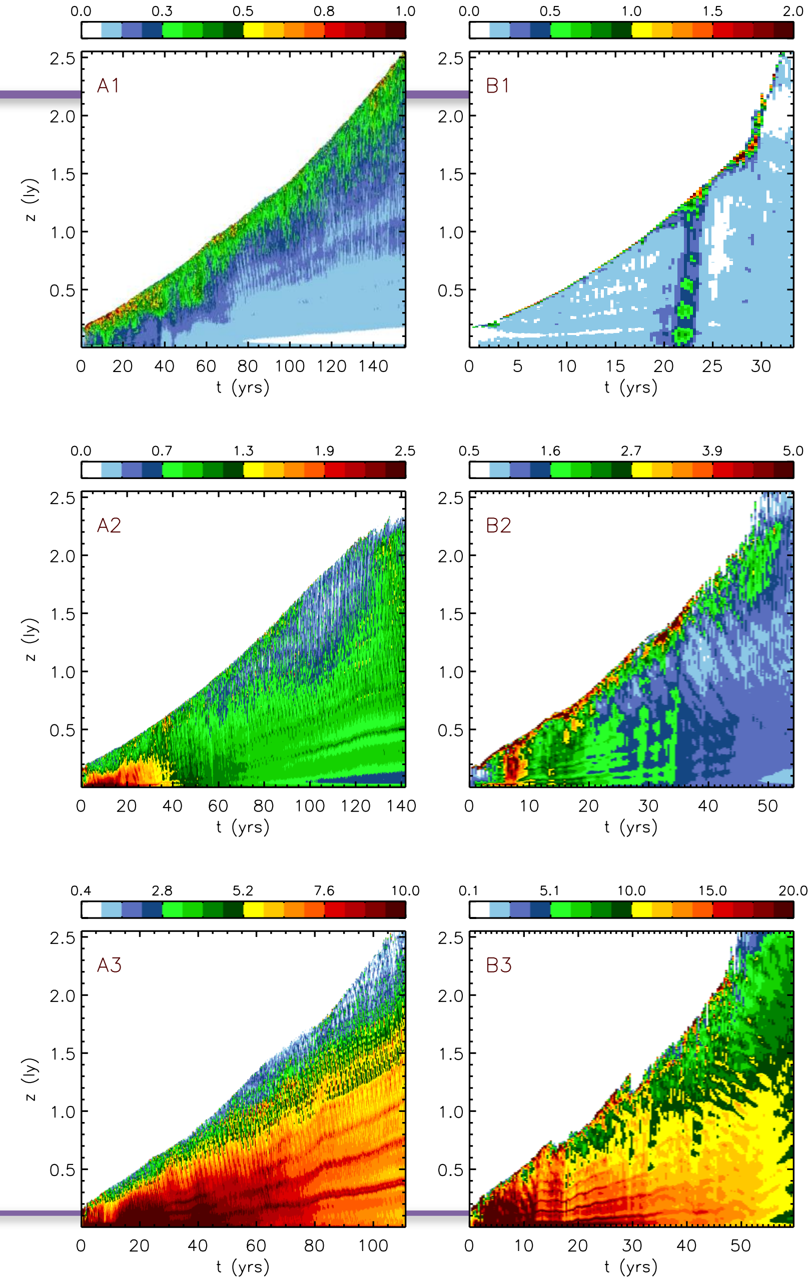
$$\bar{E}_{\rm em}(t,z) = \left\langle \frac{\boldsymbol{B}^2+\boldsymbol{E}^2}{2}, \chi \right\rangle$$

$$\bar{E}_{\rm kin}(t,z) = \langle \rho\gamma(\gamma-1), \chi \rangle$$

- Evolution of the horizontally-averaged magnetic and kinetic energy, and σ
- Periodic oscillations due to jet pinching and shock formation
- γ peaks upstream of shocks, where em and kin energies are smaller, and drops downstream

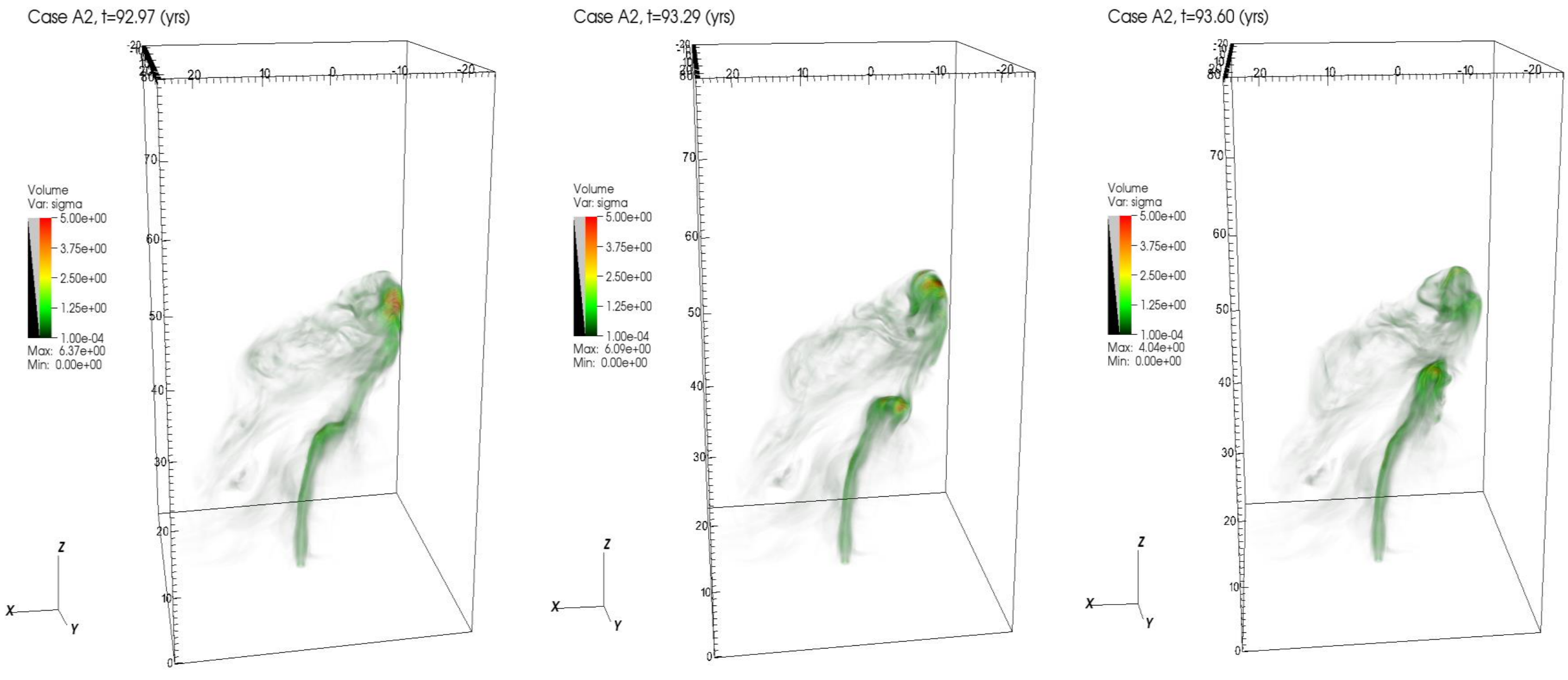


Evolution of the horizontally-averaged $\sigma = \langle B \rangle^2 / \rho\gamma^2$



Magnetic energy *vs* kinetic energy

- Short time scale evolution



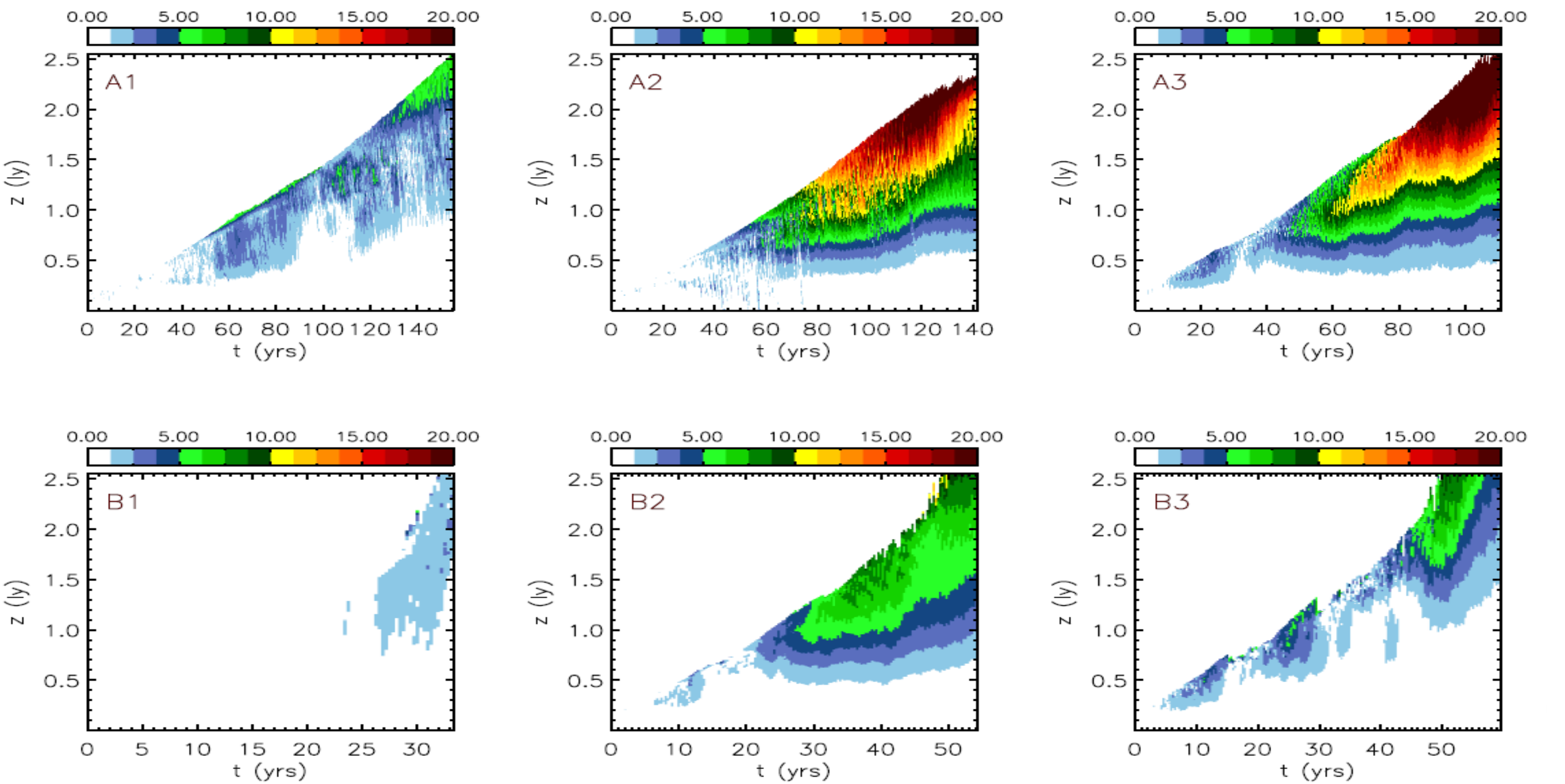
Jet Deflections

- Deflection is quantified using the jet baricenter:

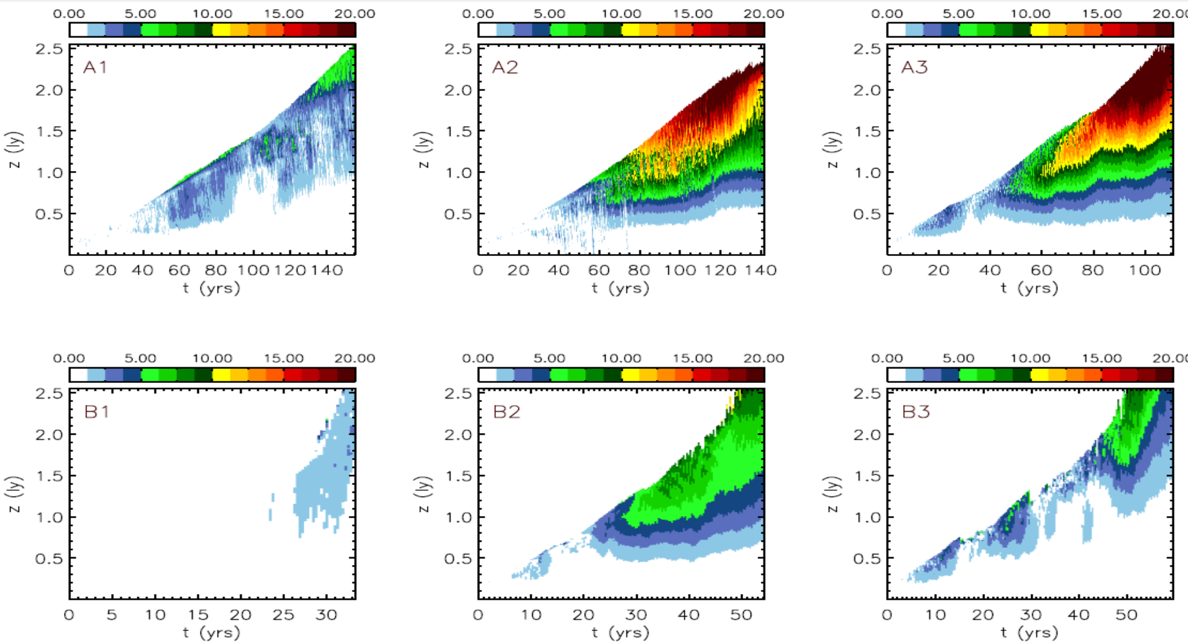
$$\bar{R}(z) \equiv \sqrt{\bar{x}^2(z) + \bar{y}^2(z)}$$

$$\bar{x}(z) = \frac{\int xQ(x,y,z)dxdy}{\int Q(x,y,z)dxdy}$$

$$\bar{y}(z) = \frac{\int yQ(x,y,z)dxdy}{\int Q(x,y,z)dxdy}$$



Jet Deflections

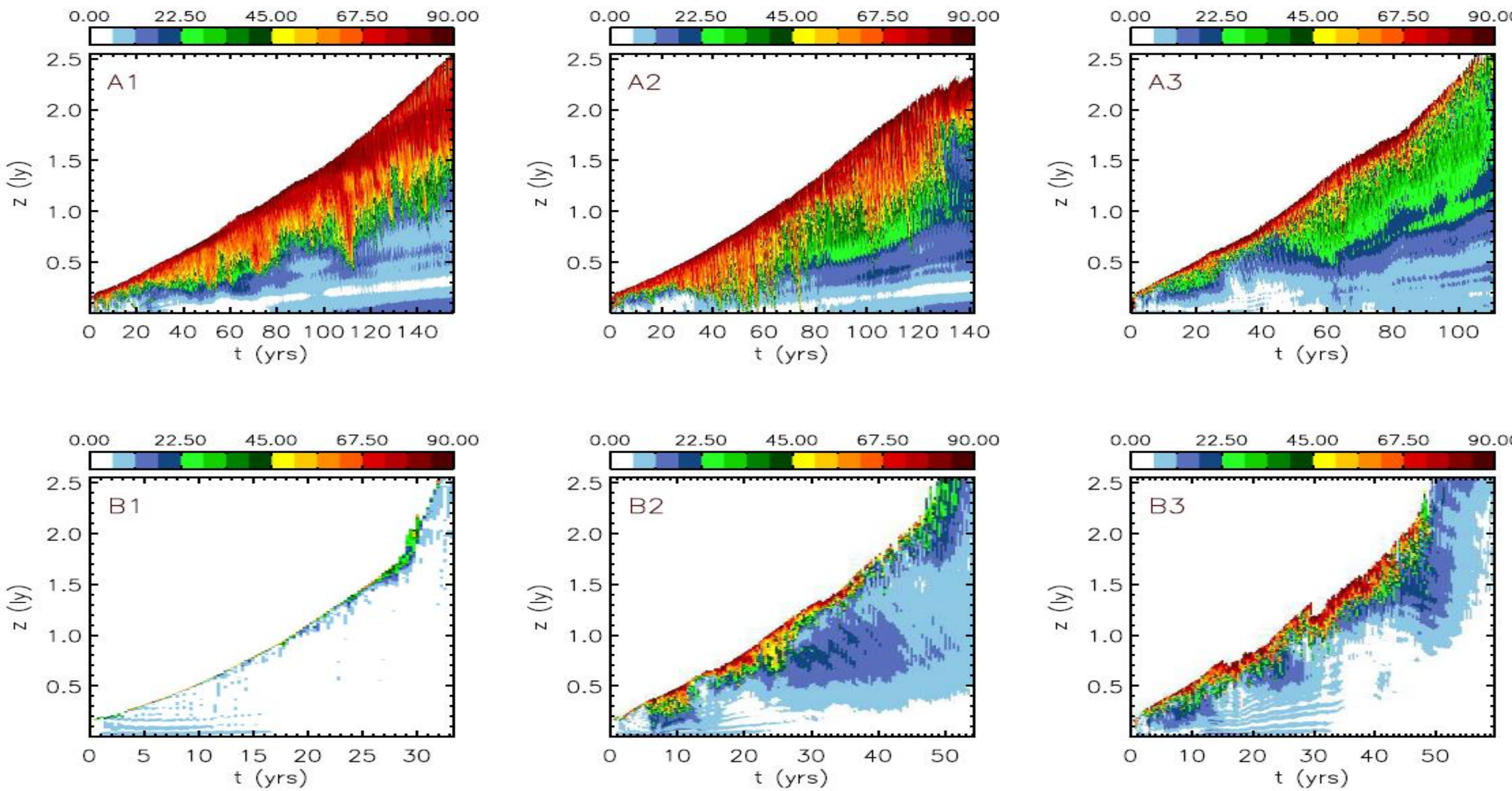


- Case A2 and A3 (low-speed, moderately/highly magnetized) jets show the largest bending (> 20 jet radii)
- Larger Lorentz factors (B2, B3) have a stabilizing effect
- Weakly magnetized jets (A1, B1) are less affected by the growth of instability

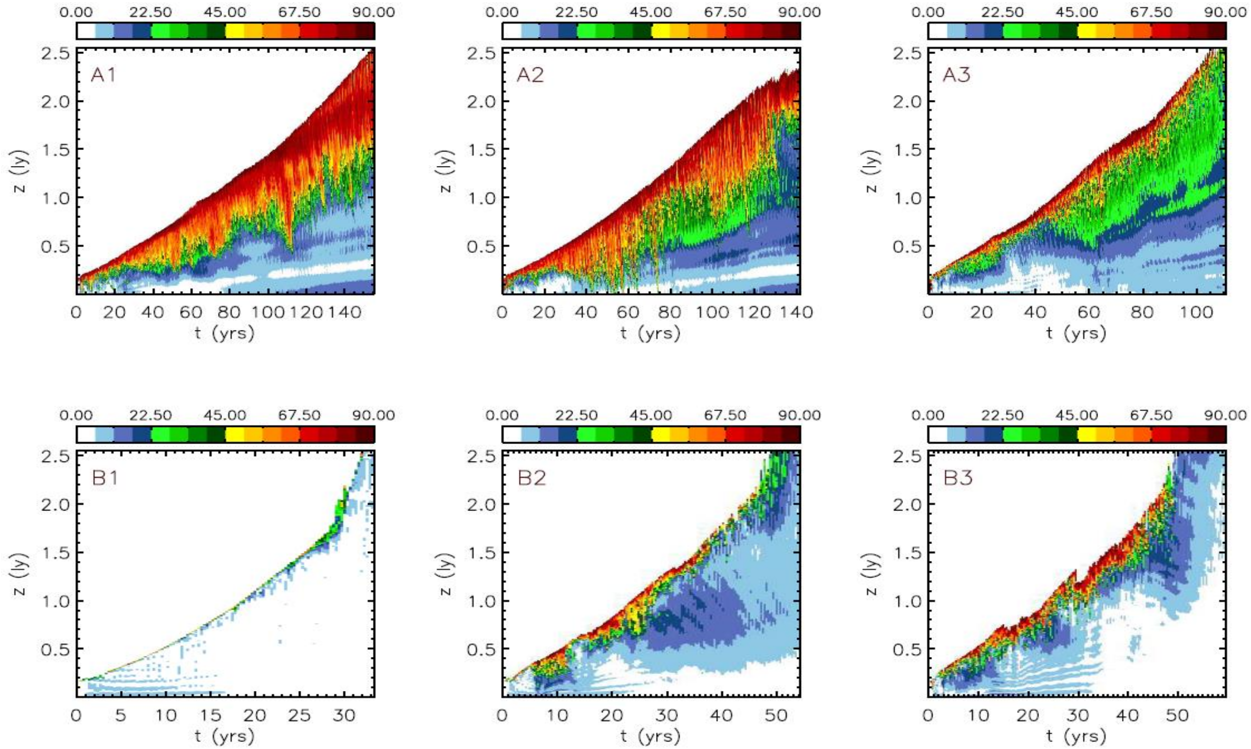
Flow Inclination

- Flow direction is measured by computing the average angle of the mass flux vector with vertical direction:

$$\bar{\theta}_{\pm}(z) = \cos^{-1}\left(\frac{\int (v \cdot \hat{z}/|v|)\chi_{\pm}\,dx\,dy}{\int \chi_{\pm}\,dx\,dy}\right)$$



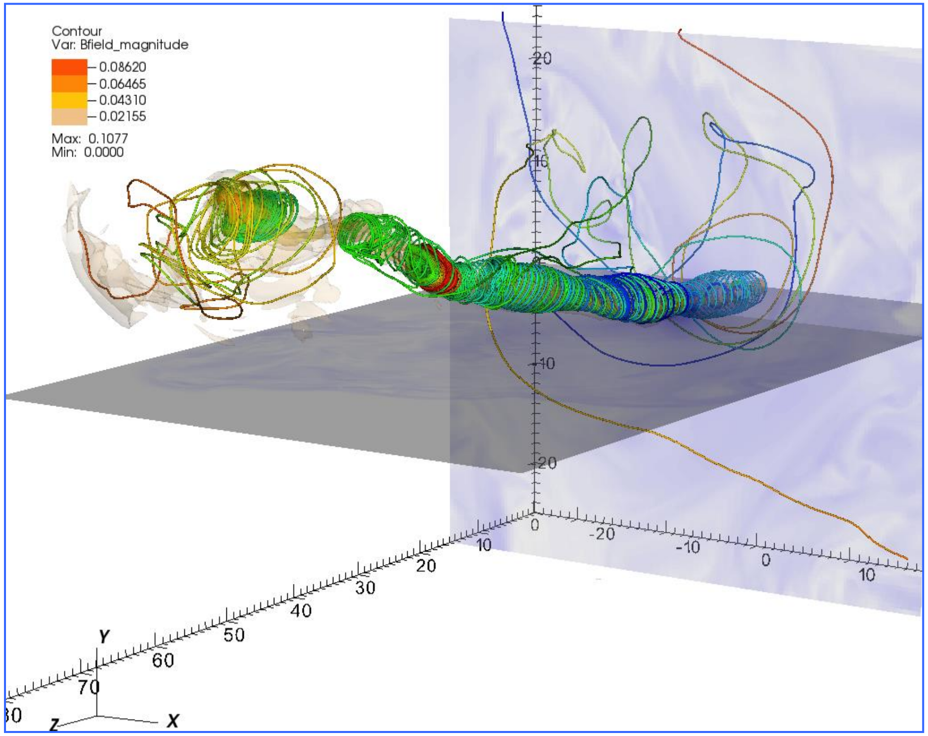
Flow Inclination



- Low-speed jets assume a large-scale curved structure
- High-speed jets more parallel and build kicks in proximity of jet's head

Magnetic Field Structure

- Magnetic field topology remains mainly toroidal or helical during the propagation
- Azimuthal field has the effect of “shielding” the core preventing interaction with the ambient
- Local pinching events along the jet rapidly evolving, reconnection
- Magnetic field dissipates and becomes turbulent in the cocoon (→ randomization)

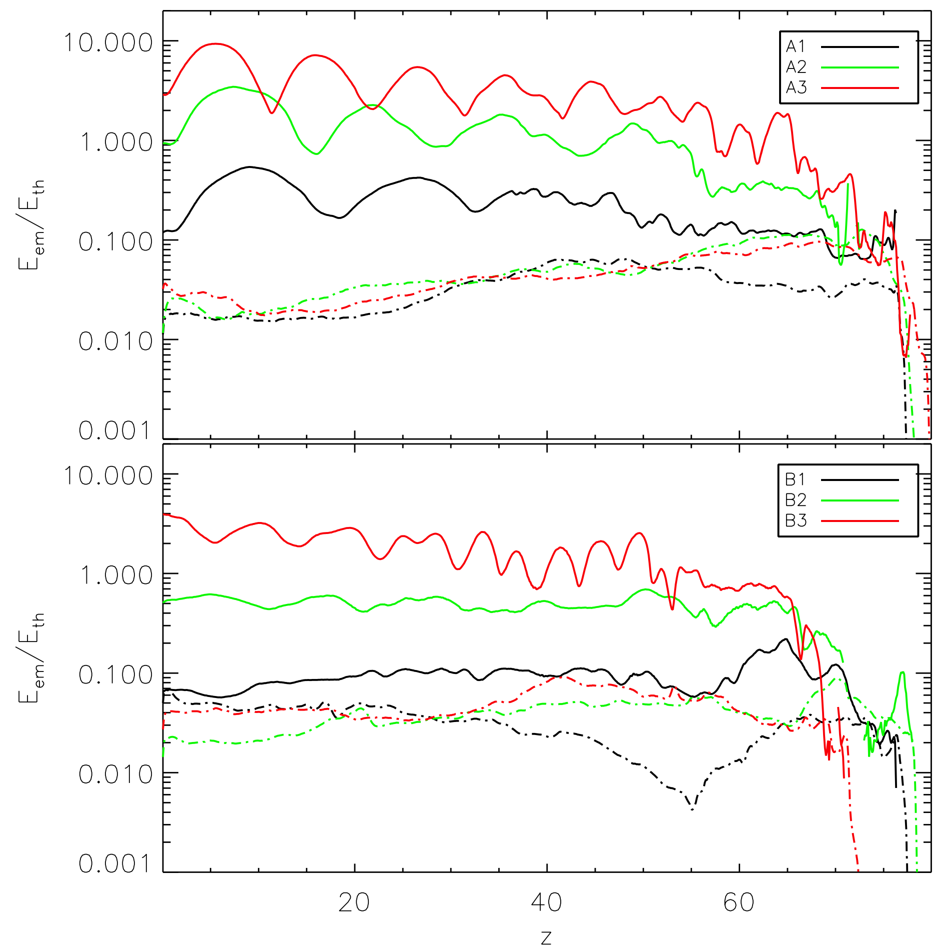


Large scale magnetic dissipation

$$\bar{E}_{\rm em,j} = \left\langle \frac{\boldsymbol{B}^2 + \boldsymbol{E}^2}{2}, \chi_j \right\rangle , \qquad \bar{E}_{\rm th,j} = \left\langle \frac{p}{\Gamma - 1}, \chi_j \right\rangle$$

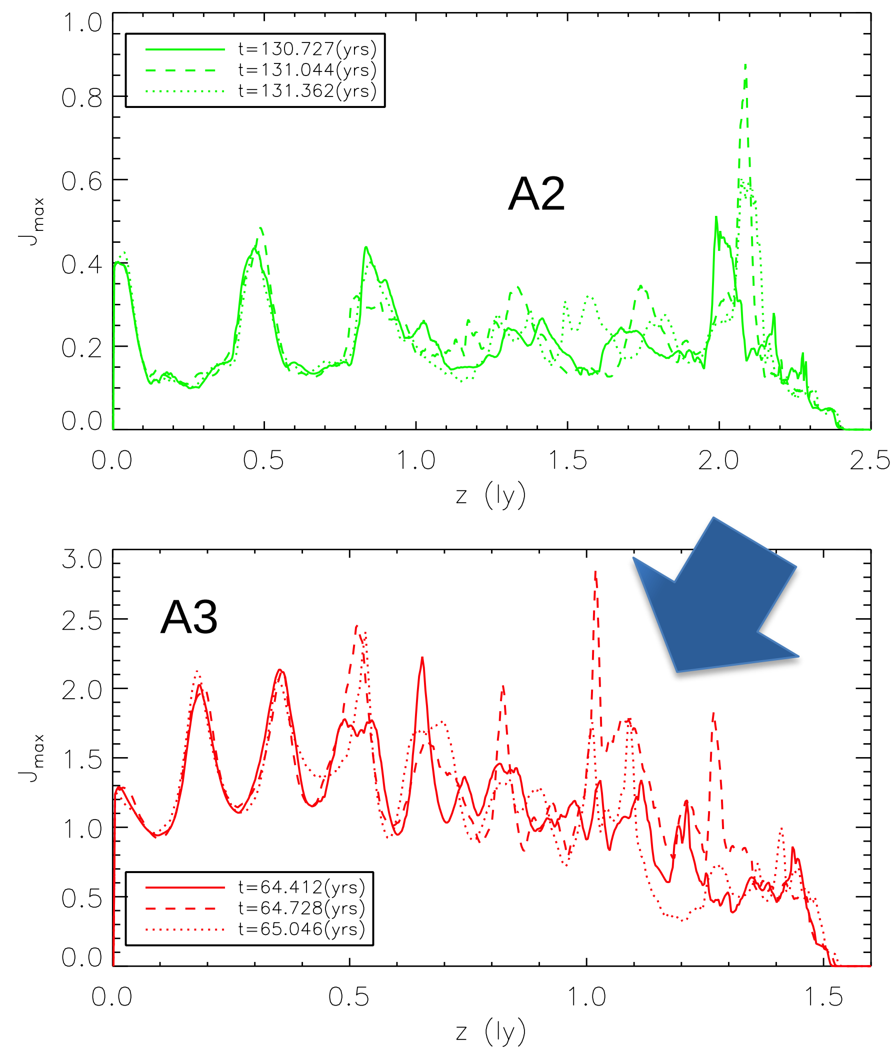
$$\bar{E}_{\rm em,e} = \left\langle \frac{\boldsymbol{B}^2 + \boldsymbol{E}^2}{2}, \chi_e \right\rangle , \qquad \bar{E}_{\rm th,e} = \left\langle \frac{p}{\Gamma - 1}, \chi_e \right\rangle$$

- Ratio between magnetic field and thermal energy inside the jet shows periodic oscillations related to the formation of conical shocks
- Outside the jet the ratio is uniform
- Drop at the termination shock
- All energy both magnetic and kinetic dispersed into the cocoon backflow

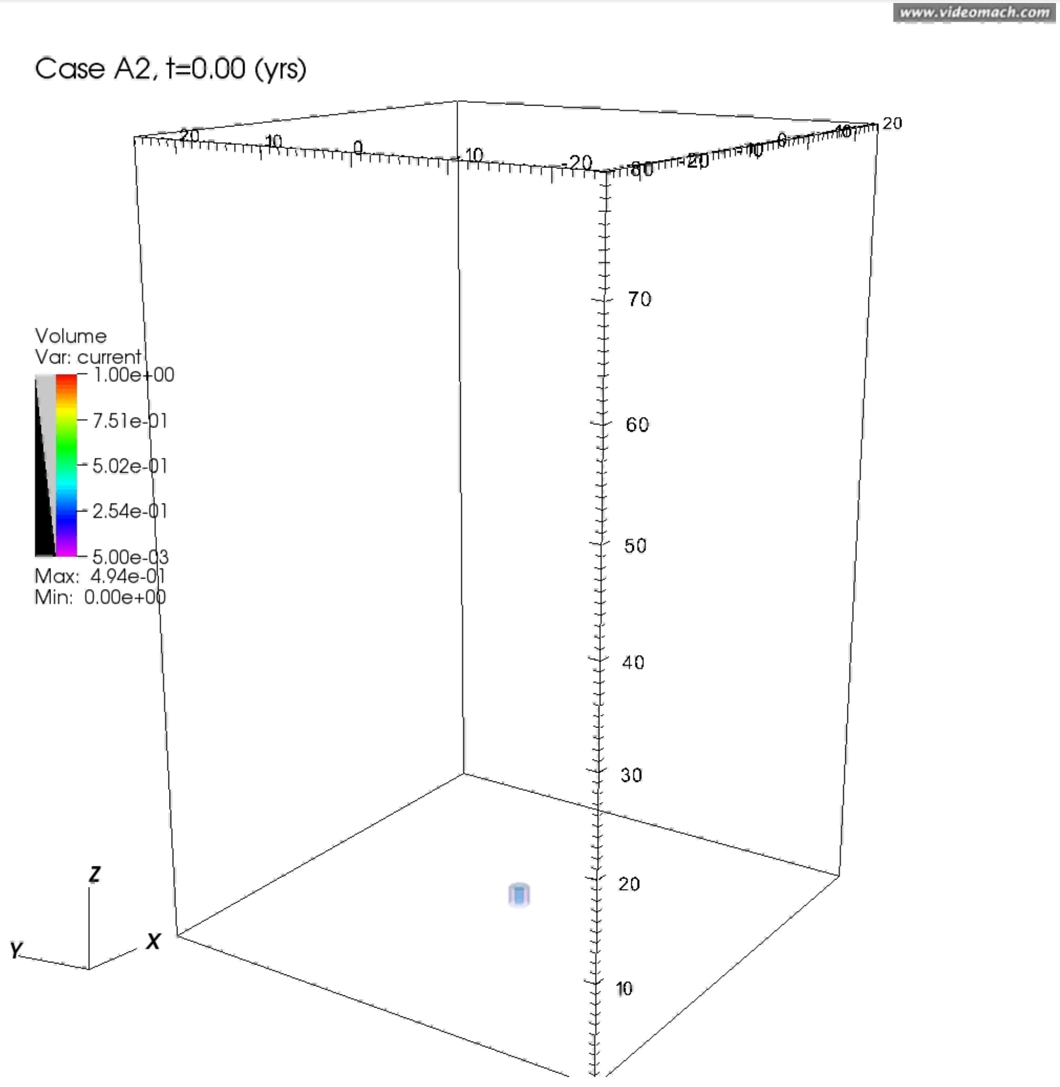


Small scale magnetic dissipation

- Maximum current density evolution over short time scales
- Rapidly evolving discontinuities in the jet's head region



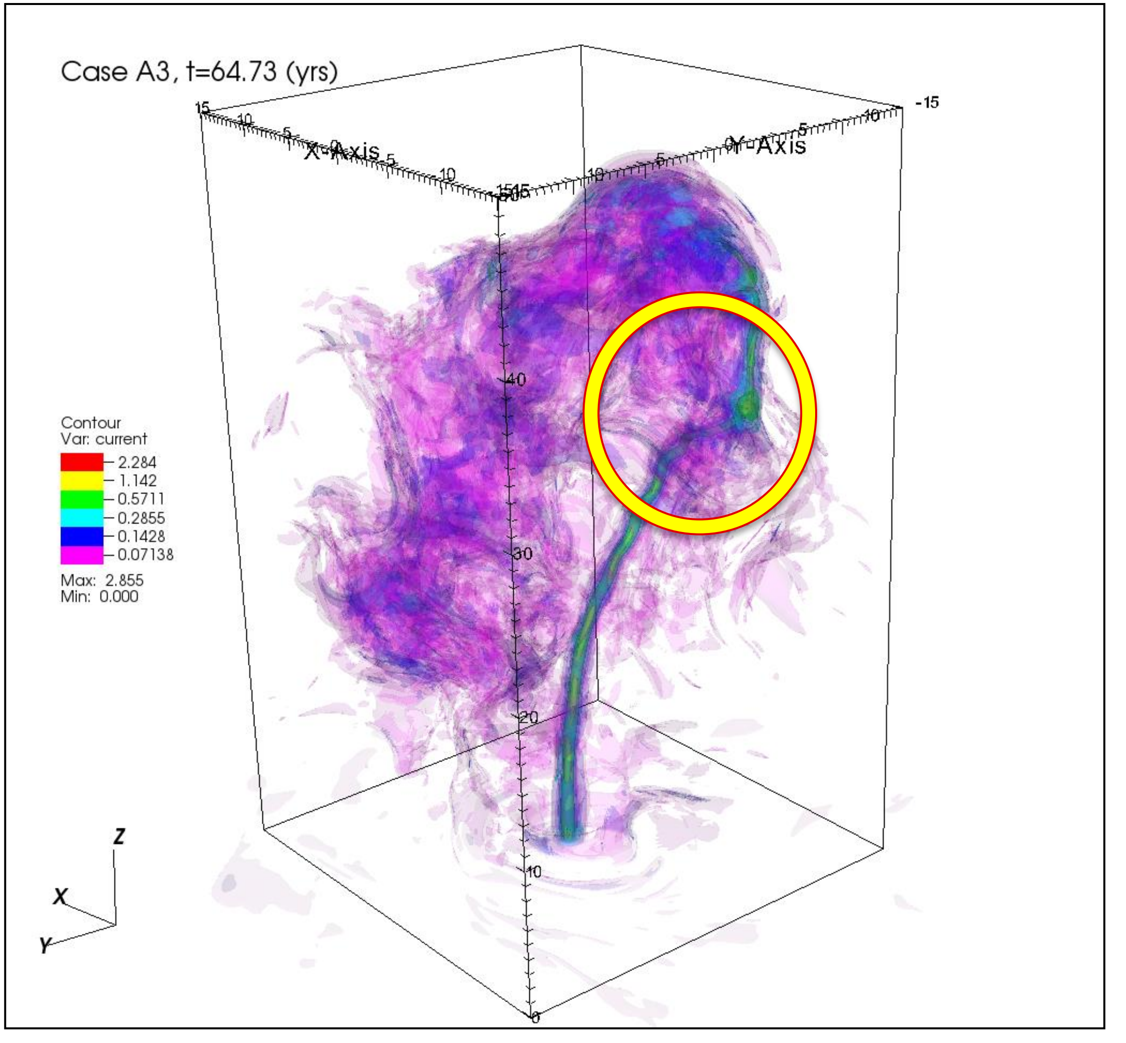
Currents



www.videomach.com
Case A2, t=0.00 (yrs)
Volume
Var: current
1.00e+00
7.51e-01
5.02e-01
2.54e-01
5.00e-03
Max: 4.94e-01
Min: 0.00e+00
70
60
50
40
30
20
10
Z
Y
X

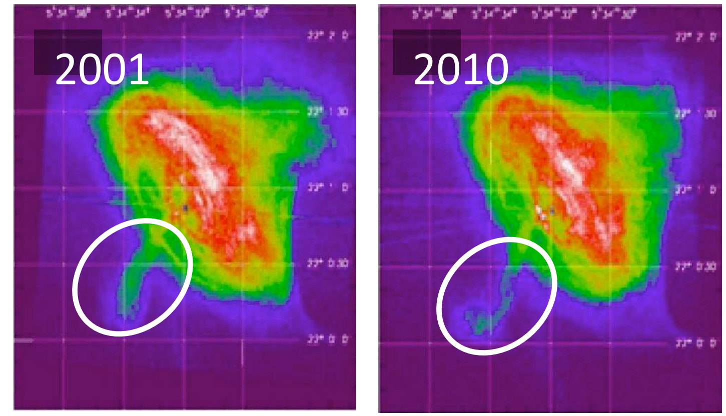
Currents and reconnection

- Evidence of explosive reconnection events in the termination region, reconnection flashes
- Warning: only numerical resistivity is present in these simulations; physical resistivity should further enhance the reconnection process

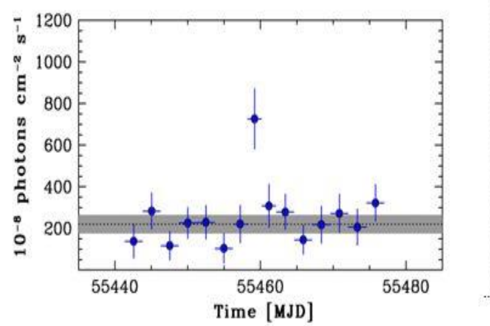


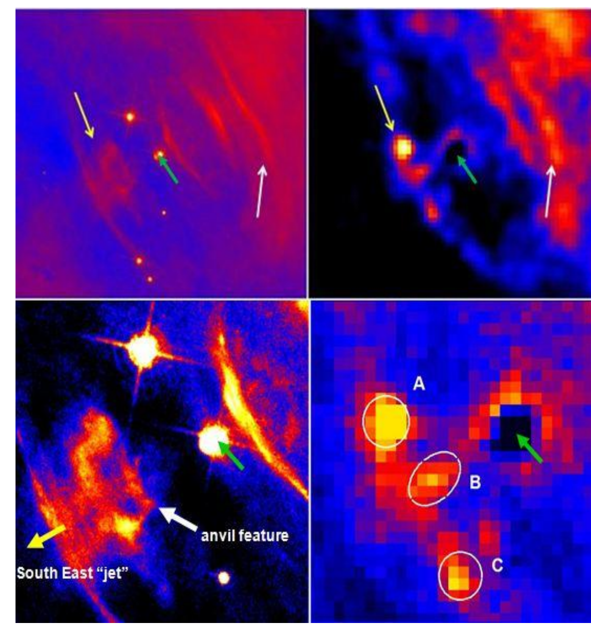
Jet Wiggling & Gamma Flares

- SE jet morphology is "S" shaped and show remarkable time variability (Weisskopf)



- Gamma flares correlated with X-ray emission variabilities in the anvil region and beyond





Summary

- 3D models of azimuthally confined relativistic jets are very different from 2D axisymmetric models:
  - Kink-unstable non-axisymmetric structures with large time-variability
  - Large σ (≥ 1) leads to considerable jet deflections
  - Pronounced asymmetric backflows
  - Jet wiggling progressively more pronounced towards the jet head
  - Multiple strong shocks are formed by change of direction
- Low-speed ($\gamma \approx 2$), moderately/highly magnetized jets ($\sigma \approx$ 1-10) are promising candidates for explaining the morphology of the Crab jet and the production of high-energy particles (*sigma-problem* solved)
- Rapid variability and reconnection events over time scales of several months/year

Future models will consider the jet-torus connection in 3D and introduce physical resistivity and radiation emission

… hopefully !